

1. 设 $f(x, y) = xy + \frac{x}{x^2 + y^2}$, 则 $f'_x(0, 1) = \underline{\hspace{2cm}}$, $f'_y(0, 1) = \underline{\hspace{2cm}}$.

答案: 2, 0

解析:

$$f(x, y) = xy + \frac{x}{x^2 + y^2},$$

$$f'_x(x, y) = y + \frac{x^2 + y^2 - 2x^2}{(x^2 + y^2)^2} = y + \frac{y^2 - x^2}{(x^2 + y^2)^2},$$

$$f'_x(0, 1) = 1 + \frac{1^2 - 0^2}{(0^2 + 1^2)^2} = 2$$

$$f'_y(x, y) = x - \frac{2yx}{(x^2 + y^2)^2}, f'_y(0, 1) = 0$$

$$2. \text{函数 } f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & x^2 + y^2 \neq 0 \\ 0 & x^2 + y^2 = 0 \end{cases} \quad (\quad).$$

A. 处处连续;

B. 处处有极限, 但不连续;

C. 仅在 (0, 0) 点连续;

D. 除 (0, 0) 点外处处连续.

答案: A

解析:

$$|f(x, y)| \leq \frac{|xy|}{\sqrt{x^2 + y^2}} \leq \sqrt{x^2 + y^2}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (\sqrt{x^2 + y^2}) = 0, \text{ 故 } \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} f(x, y) = 0, \text{ 选 A}$$

$$3. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{3xy}{x^2 + y^2} = (\quad).$$

A. $\frac{3}{2}$;

B. 0;

C. $\frac{6}{5}$;

D. 不存在.

答案: D

解析:

$$\lim_{\substack{x \rightarrow 0 \\ y=x}} \frac{3xy}{x^2 + y^2} = \lim_{\substack{x \rightarrow 0 \\ y=x}} \frac{3x^2}{x^2 + x^2} = \frac{3}{2}, \lim_{\substack{x \rightarrow 0 \\ y=2x}} \frac{3xy}{x^2 + y^2} = \lim_{\substack{x \rightarrow 0 \\ y=x}} \frac{6x^2}{x^2 + 4x^2} = \frac{6}{5}$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{3xy}{x^2 + y^2} \text{ 不存在, 选 D}$$

4. 函数 $f(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x} & xy \neq 0 \\ 0 & xy = 0 \end{cases}$, 则极限 $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} f(x, y) = (\quad)$.

A. 不存在; B. 1; C. 0 D. 2.

答案: C

解析:

$$|f(x, y)| \leq |x| + |y|$$

$$\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (|x| + |y|) = 0, \text{ 故 } \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} f(x, y) = 0, \text{ 选 C}$$

5. 设函数 $u = \ln(1 + x^2 + y^2 + z^2)$, 求 $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \Big|_{(1,1,1)}$.

答案:

$$\begin{aligned} & \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \Big|_{(1,1,1)} \\ &= \left(\frac{2x}{1+x^2+y^2+z^2} + \frac{2y}{1+x^2+y^2+z^2} + \frac{2z}{1+x^2+y^2+z^2} \right) \Big|_{(1,1,1)} \\ &= \frac{2+2+2}{1+1^2+1^2+1^2} = \frac{6}{4} = \frac{3}{2} \end{aligned}$$