

1. 已知向量  $\mathbf{a} = (2, 3, -4)$ ,  $\mathbf{b} = (5, -1, 1)$ , 则向量  $\mathbf{c} = 2\mathbf{a} - 3\mathbf{b}$  在  $y$  轴上的分量是\_\_\_\_\_.

答案:  $9j$

解析:

$$\vec{a} = (2, 3, -4), \vec{b} = (5, -1, 1)$$

$$2\vec{a} - 3\vec{b} = (4, 6, -8) - (15, -3, 3) = (-11, 9, -11)$$

$y$ 轴上的分量为  $9\vec{j}$ , 投影为9

2. 已知  $|\vec{a}| = 5$ ,  $|\vec{b}| = 8$ ,  $(\vec{a}, \vec{b}) = \frac{\pi}{3}$ , 则  $|\vec{a} - \vec{b}| =$ \_\_\_\_\_.

答案: 7

解析:

$$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}$$

$$= 5^2 + 8^2 - 2(5 \times 8 \times \frac{1}{2}) = 25 + 64 - 40 = 49$$

$$\Rightarrow |\vec{a} - \vec{b}| = 7$$

3. 平行于  $x$  轴且过两点  $(4, 0, -2)$  和  $(5, 1, 7)$  的平面方程为\_\_\_\_\_.

答案:  $9y - z - 2 = 0$

解析:

设平面方程为  $By + Cz + D = 0$

将  $(4, 0, -2)$ ,  $(5, 1, 7)$  代入方程得

$$\begin{cases} -2C + D = 0 \\ B + 7C + D = 0 \end{cases} \text{解得 } D = 2C, B = -9C$$

故  $-9Cy + Cz + 2C = 0$ , 即

$$9y - z - 2 = 0$$

4. 设一平面过原点和点  $(6, -3, 2)$ , 且与平面  $4x - y + 2z = 8$  垂直, 则该平面

方程为\_\_\_\_\_

答案:  $2x + 2y - 3z = 0$ .

解析:

设  $\pi: Ax + By + Cz = 0, \vec{n} = (A, B, C)$

$\pi_1: 4x - y + 2z - 8 = 0, \vec{n}_1 = (4, -1, 2)$

$\pi \perp \pi_1, \vec{n} \cdot \vec{n}_1 = 4A - B + 2C = 0,$

$\pi$  过点  $(6, -3, 2), 6A - 3B + 2C = 0$

解得  $A = B, C = -\frac{3B}{2}, Ax + Ay - \frac{3}{2}Az = 0,$

$x + y - \frac{3}{2}z = 0,$  即  $2x + 2y - 3z = 0$

5. 过  $Oz$  轴, 且与平面  $2x + y - 3z - 1 = 0$  垂直的平面方程为\_\_\_\_\_.

答案:  $x - 2y = 0$

解析:

设  $\pi: Ax + By = 0, \vec{n} = (A, B, 0)$

$\pi_1: 2x + y - 3z - 1 = 0, \vec{n}_1 = (2, 1, -3)$

$\pi \perp \pi_1, \vec{n} \cdot \vec{n}_1 = 2A + B = 0,$

解得  $-2A = B, Ax - 2Ay = 0,$

$x - 2y = 0$

6. 设一平面与向量  $\vec{a} = (2, 1, -1)$  平行, 且在  $Ox$  轴,  $Oy$  轴, 上的截距依次为 3 和 -2, 则该平面方程为\_\_\_\_\_.

答案:  $2x - 3y + z - 6 = 0$

解析:

设  $\pi: \frac{x}{3} + \frac{y}{-2} + \frac{z}{c} = 1, \vec{n} = (\frac{1}{3}, -\frac{1}{2}, \frac{1}{c})$

$\vec{a} = (2, 1, -1)$

$\pi // \vec{a}, \vec{n} \perp \vec{a}, \vec{n} \cdot \vec{a} = \frac{2}{3} - \frac{1}{2} - \frac{1}{c} = 0,$

解得  $c = 6, \pi: \frac{x}{3} + \frac{y}{-2} + \frac{z}{6} = 1$  即

$2x - 3y + z = 6$

7. 过点  $M(1, 2, -1)$ , 且与直线  $L: x = -t + 2, y = 3t - 4, z = t - 1$  垂直的平面方程为\_\_\_\_\_

答案:  $-x + 3y + z - 4 = 0$

解析:

$L: x = -t + 2, y = 3t - 4, z = t - 1, \vec{s} = (-1, 3, 1)$

$\pi \perp L, \vec{n} = \vec{s} = (-1, 3, 1), -(x-1) + 3(y-2) + (z+1) = 0$

即  $-x + 3y + z - 4 = 0$

$$-x+3y+z-4=0$$

8.空间三点  $A(1,-1,2)$  ,  $B(4,5,4)$  ,  $C(2,2,2)$  , 求三角形  $ABC$  的面积.

答案:

$$\overrightarrow{AB} = (4-1, 5+1, 4-2) = (3, 6, 2)$$

$$\overrightarrow{AC} = (2-1, 2+1, 2-2) = (1, 3, 0)$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 6 & 2 \\ 1 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 3 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & 6 \\ 1 & 3 \end{vmatrix} \vec{k}$$

$$= -6\vec{i} + 2\vec{j} + 3\vec{k}$$

$$S_{\triangle ABC} = \frac{1}{2} |\overrightarrow{AB}| |\overrightarrow{AC}| \sin(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$= \frac{1}{2} \sqrt{6^2 + 2^2 + 3^2} = \frac{7}{2}$$