

1. 设 L 是正向圆周 $x^2 + y^2 = 1$, 计算曲线积分 $\oint_L e^{x^2-y^2} (x dx + y dy) =$ _____ .

答案: 0

2. 设 L 为任意一条分段光滑的闭曲线, 则曲线积分 $\oint_L (2xy - 2x) dx + (x^2 - 4y) dy =$ _____.

答案: 0

解析:

$$P(x, y) = 2xy - 2x, Q(x, y) = x^2 - 4y$$

$$\frac{\partial Q}{\partial x} = 2x, \frac{\partial P}{\partial y} = 2x, \text{格林公式}$$

$$\oint_L (2xy - 2x) dx + (x^2 - 4y) dy = \iint_D (2x - 2x) dx dy = 0$$

3. 单连通域 G 内的函数 $P(x, y)$, $Q(x, y)$ 具有一阶连续偏导数, 则 $\int_C P dx + Q dy$ 在 G 内与路径无关的充要条件是在 G 内恒有 ().

$$\text{A. } \frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y} = 0; \quad \text{B. } \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0;$$

$$\text{C. } \frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y} = 0; \quad \text{D. } \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 .$$

答案: B

$$\text{解析: } \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Rightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$$

4. 求 $\int_L y dx$, 其中 L 是直线 $x = 0, y = 0, x = 2$ 和 $y = 4$ 所组成的矩形边界, 逆时针.

答案:

$$L = L_1 + L_2 + L_3 + L_4$$

$$L_1: y = 0, x: 0 \rightarrow 2, L_2: x = 2, y: 0 \rightarrow 4,$$

$$L_3: y = 4, x: 2 \rightarrow 0, L_4: x = 0, y: 4 \rightarrow 0,$$

$$\int_{L_1} y dx = 0, \int_{L_2} y dx = 0, \int_{L_3} y dx = \int_2^0 4 dx = -8, \int_{L_4} y dx = 0$$

$$\int_L y dx = \int_{L_1} y dx + \int_{L_2} y dx + \int_{L_3} y dx + \int_{L_4} y dx = -8$$