

1. $z = f(x, y)$ 在有界闭区域 D 上连续是二重积分存在的 ().

- A. 充分非必要条件; B. 必要非充分条件;
C. 充分必要条件; D. 既非充分又非必要条件.

答案: A

2. 把二重积分 $\iint_D e^{-x^2-y^2} dx dy$ 在极坐标系中化为二次积分, 其中 D 由 $x^2 + y^2 \leq 1$ 所围成 ().

- A. $\int_0^{2\pi} d\theta \int_0^1 e^{-r^2} dr$; B. $4 \int_0^{\frac{\pi}{2}} d\theta \int_0^1 e^{-r^2} dr$;
C. $2 \int_0^{\frac{\pi}{2}} d\theta \int_0^1 r e^{-r^2} dr$; D. $\int_0^{2\pi} d\theta \int_0^1 r e^{-r^2} dr$.

答案: D

解析:

$$D = \{(x, y) | x^2 + y^2 \leq 1\} = \{(r, \theta) | 0 \leq \theta \leq 2\pi, 0 \leq r \leq 1\}$$

$$d\sigma = r dr d\theta,$$

$$\iint_D e^{-x^2-y^2} dx dy = \int_0^{2\pi} d\theta \int_0^1 r e^{-r^2} dr$$

3. 设平面区域 $D: 1 \leq x^2 + y^2 \leq 4$, $f(x, y)$ 是在区域 D 上的连续函数, 则 $\iint_D f(\sqrt{x^2 + y^2}) dx dy$ 等于 ().

- A. $2\pi \int_1^2 r f(r) dr$; B. $2\pi \left[\int_0^2 r f(r) dr + \int_0^1 r f(r) dr \right]$;
C. $2\pi \int_1^2 r f(r^2) dr$; D. $2\pi \left[\int_0^2 r f(r^2) dr + \int_0^1 r f(r^2) dr \right]$

答案: A

解析:

$$D = \{(x, y) | 1 \leq x^2 + y^2 \leq 4\} = \{(r, \theta) | 0 \leq \theta \leq 2\pi, 1 \leq r \leq 2\}$$

$$d\sigma = dx dy = r dr d\theta, \sqrt{x^2 + y^2} = r$$

$$\iint_D f(\sqrt{x^2 + y^2}) dx dy = \int_0^{2\pi} d\theta \int_1^2 r \cdot f(r) dr = 2\pi \int_1^2 r f(r) dr$$

4. 设 $I_1 = \iint_{D_1} (x^2 + y^2) d\sigma$, 其中 D_1 是矩形闭区域: $-1 \leq x \leq 1, -2 \leq y \leq 2$, 又

$I_2 = \iint_{D_2} (x^2 + y^2) d\sigma$, 其中 D_2 是矩形闭区域: $0 \leq x \leq 1, 0 \leq y \leq 2$, 利用二重积分的几何意义

说明 I_1 与 I_2 之间的关系 ().

- (A) $I_1 = 3I_2$; (B) $I_1 = 2I_2$; (C) $I_1 = I_2$; (D) $I_1 = 4I_2$.

答案: D

5. 设 $D: 1 \leq x^2 + y^2 \leq 4$, 则 $\iint_D \sqrt{x^2 + y^2} dx dy = (C)$.

A. $\int_0^{2\pi} d\theta \int_0^1 r^2 dr$; B. $\int_0^{2\pi} d\theta \int_1^4 r^2 dr$; C. $\int_0^{2\pi} d\theta \int_1^2 r^2 dr$; D. $\int_0^{2\pi} d\theta \int_1^2 r dr$.

答案: C

解析:

$$D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 4\} = \{(r, \theta) \mid 0 \leq \theta \leq 2\pi, 1 \leq r \leq 2\}$$

$$d\sigma = dx dy = r dr d\theta, \sqrt{x^2 + y^2} = r$$

$$\iint_D \sqrt{x^2 + y^2} dx dy = \int_0^{2\pi} d\theta \int_1^2 r \cdot r dr = \int_0^{2\pi} d\theta \int_1^2 r^2 dr$$

6. 计算 $\iint_D (3x + 2y) d\sigma$, 其中 D 是由两坐标轴及直线 $x + y = 2$ 所围成的闭区域.

答案: $\iint_D (3x + 2y) d\sigma = \int_0^2 dx \int_0^{2-x} (3x + 2y) dy$

$$= \int_0^2 [3xy + y^2]_0^{2-x} dx = \int_0^2 (4 + 2x - 2x^2) dx = \left[4x + x^2 - \frac{2}{3}x^3 \right]_0^2 = \frac{20}{3}.$$